2/26/18 "Mistakes are the portal of Discovery." - James Joyce

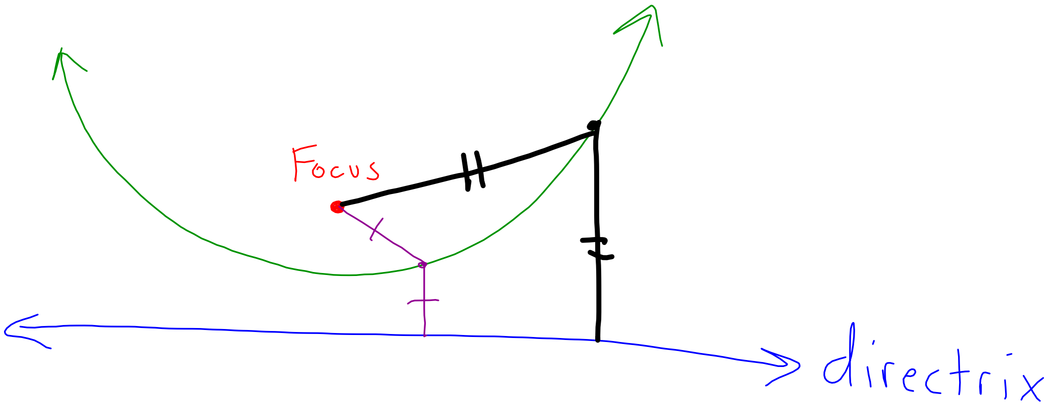
HW: "Locus Definition of a Parabola" Homework section

AIM: What is a Parabola?

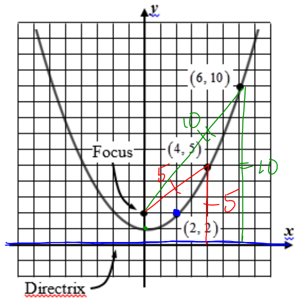
Warm Up:

THE LOCUS DEFINITION OF A PARABOLA

A parabola is the collection of all points **equidistant** from a fixed point (known as its **focus**) and a fixed line (known as its **directrix**).



***Exercise* #1:** The parabola $y=\frac{1}{4}x^2+1$ is shown graphed below with selected points shown. For this parabola, its focus is the point $(0, 2)$ and its directrix is the x-axis.



(a) How far is the turning point (vertex) $(0,1)$ from both the focus and directrix? How far is the point $(2, 2)$ from both?

Vertex is 1 unit from the focus and directrix.

(2,2) is 2 units away from both.

(b) Use the distance formula to verify that the point $(4, 5)$ is the same distance away from the focus and directrix. Draw line segments from the focus and directrix to this point to visualize the distance. Repeat for the point $(6, 10)$

$$\text{distance} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Focus: (0,2) Point (4,5)

$$d=\sqrt{(4-0)^2+(5-2)^2}$$
$$=\sqrt{16+9}$$
$$=\sqrt{25}$$
$$\boxed{d=5}$$

Focus (0,2) Point (6,10)

$$d=\sqrt{(0-6)^2+(2-10)^2}$$
$$=\sqrt{36+64}$$
$$=\sqrt{100}$$
$$\boxed{d=10}$$

(c) Use the distance formula to show that the equation of this parabola is $y=\frac{1}{4}x^2+1$ based on the locus definition of a parabola.

Focus: (0,2) Point: (x,y)

Distance from any point (x,y) on parabola to the directrix (x-axis in this case) is the y-value

Parabola

Distance to Directrix = Distance to Focus

$$y=\sqrt{(0-x)^2+(2-y)^2}$$

$$y=\sqrt{(-x)^2+(2-y)^2}$$

$$(2-y)^2=(2-y)(2-y)$$
$$=4-2y-2y+y^2$$
$$=y^2-4y+4$$

$$(y)^2=\left(\sqrt{x^2+y^2-4y+4}\right)^2$$

$$y^2=x^2+y^2-4y+4$$
$$-y^2 \qquad -y^2$$

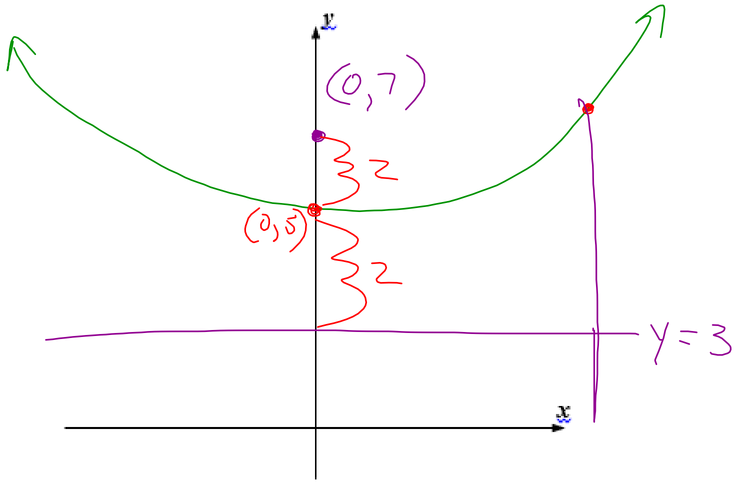
$$0=x^2-4y+4$$
$$+4y \qquad +4y$$

$$\frac{4y}{4}=\frac{x^2}{4}+\frac{4}{4}$$

$$\boxed{y=\frac{1}{4}x^2+1}$$

***Exercise* #2:** Consider a parabola whose focus is the point $(0, 7)$ and whose directrix is the line $y = 3$.

(a) Sketch a diagram of the parabola below and identify its turning point.



(b) Determine the equation of the parabola using the locus definition.

Focus: (0,7)

Directrix y=3

Distance to Directrix = Distance to Focus

$$y - 3 = \sqrt{(0-x)^2 + (7-y)^2}$$

$$(y-3)^2 = \left(\sqrt{x^2 + y^2 - 14y + 49}\right)^2$$

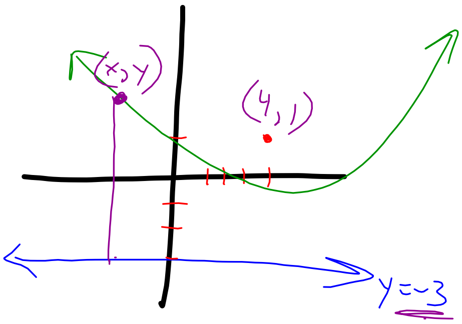
$$(7-y)(7-y)$$
$$49 - 7y - 7y + y^2$$
$$y^2 - 14y + 49$$

$$y^2 - 6y + 9 = x^2 + y^2 - 14y + 49$$
$$-y^2 + 14y - 9 \qquad -y^2 + 14y - 9$$

$$\frac{8y}{8} = \frac{x^2}{8} + \frac{40}{8}$$

$$y = \frac{1}{8}x^2 + 5$$

***Exercise* #3:** Determine the equation of the parabola whose focus is the point $(4,1)$ and whose directrix is the horizontal line $y=-3$. First, draw a diagram that shows the parabola, then carefully use the distance formula to derive its equation.



Dist to directrix = Distance to Focus

$$y-(-3) = \sqrt{(x-4)^2+(y-1)^2}$$

$$(y+3)^2 = \left(\sqrt{(x-4)^2+(y-1)^2}\right)^2$$

$$y^2+6y+9 = (x-4)^2+(y-1)^2$$

$$y^2+6y+9 = x^2-8x+16+y^2-2y+1$$

$$-y^2+2y \qquad\qquad -y^2+2y$$

$$8y+9 = x^2-8x+17$$

$$-9 \qquad\qquad -9$$

$$\frac{8y}{8} = \frac{x^2}{8}-\frac{8x}{8}+\frac{8}{8}$$

$$y=\frac{1}{8}x^2-1x+1$$

Equation of Parabolas

1) Distance to Directrix = Distance to Focus

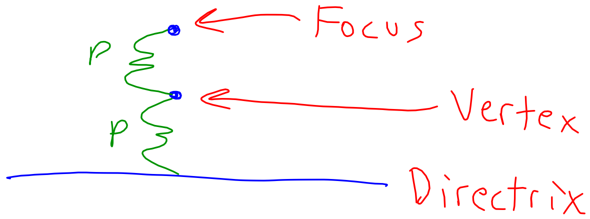
$y - \left(\begin{matrix}\text{equation} \\ \text{for Directrix}\end{matrix}\right) =$ Distance formula from (x, y) to Focus Point

2) $$y - k = \frac{(x-h)^2}{4p}$$

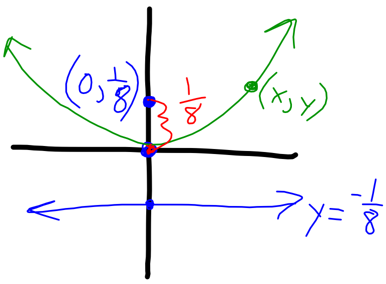
Vertex is (h, k)

p is Distance from Vertex to Focus

(*) Vertex is directly between the Focus and Directrix



1) Find equation of Parabola that has a focus of $(0, \frac{1}{8})$ and vertex @ origin $(0,0)$.



Method 1:

$$y + \frac{1}{8} = \sqrt{(x-0)^2 + (y - \frac{1}{8})^2}$$

Method 2:

$$y - k = \frac{(x-h)^2}{4p}$$

$(h,k) = (0,0)$ $p = \frac{1}{8}$

$$y - 0 = \frac{(x-0)^2}{4(\frac{1}{8})}$$

$$y = \frac{x^2}{1/2}$$

$$y = \frac{2}{1}x^2 = \boxed{y = 2x^2}$$